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Modelling and experiment of railway ballast vibrations

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Abstract

The vibration of railway ballast is a key factor to cause track geometry change and increase of track maintenance costs. So far the methods for analyzing and testing the vibration of the granular ballast have not been well formed. In this paper, a five-parameter model for analysis of the ballast vibration is established based upon the hypothesis that the load-transmission from a sleeper to the ballast approximately coincides with the cone distribution. The concepts of shear stiffness and shear damping of the ballast are introduced in the model in order to consider the continuity of the interlocking ballast granules. A full-scale field experiment is carried out to measure the ballast acceleration excited by moving trains. Theoretical simulation results agree well with the measured results. Hence the proposed ballast vibration model has been validated.

1. Introduction

Ballast is an important component of railway track structures. The vibration of ballast causes ballast settlement and the change of track geometries, therefore influences the track maintenance costs. According to the statistic of Chinese Railways (CR) [1], about 75% of the daily maintenance work on track structures is due to the ballast and its deformation. Thus, it is very important to investigate the characteristics of the ballast vibration so as to minimize its vibration level.

However, it is difficult to analyze the vibration of ballast because of its granular configuration and special mechanism of action. So far, little research work has been done on dynamic modelling of the ballast as a continuum so that its behavior can be predicted when excited by moving loaded wheels [2]. The dynamic effect of the ballast is usually considered by a series of springs and

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dampers under the sleepers in the analyses of track dynamics [3]. The representative theoretical work on the ballast vibration is the hypothesis of the load being transmitted within a cone region in the ballast by Ahlbeck et al. [4], and the test of ballast accelerations carried out by Sato et al. [5] could be regarded as a representation of experimental work. In 1993, Zhai et al. [6] established a detailed model for investigating dynamic interactions between vehicles and tracks, in which Ahlbeck's hypothesis was adopted to establish the ballast vibration model. Ripke et al. [7] and Oscarsson et al. [8,9] then used this mathematical model in their simulations of train/track interactions.

Although the model provides a possibility to analyze the vibration of ballast, the model has not been validated by experiments. The purpose of this paper is on one hand to improve the ballast vibration model and the method to determine its parameters, and on the other hand to validate the theoretical model by field experiments.

2. Model for analysis of ballast vibration

Ahlbeck [4] assumed that the load transmitting from a sleeper to the ballast approximately coincides with the cone distribution. That is to say, the stresses of the ballast are uniformly distributed over the cone region and zero outside the cone. The inclination of the cone is just the ballast stress pervasion angle corresponding to the Poisson's ratio [4]. Thus the effective acting region of the ballast under each sleeper can be determined, as shown in Fig. 1.

On the basis of this assumption, it could be concluded that the vibrating part of the ballast under each sleeper is just the cone region such as shown in Fig. 2. Therefore, the continuous granular ballast could be modelled as a series of separate vibrating masses when analyzing the track dynamics, by which the analytical process of the ballast vibration is greatly simplified. In fact, Rücker et al. [10] has already concluded that the theoretical investigation on ballast vibrations is extremely difficult and impossible to be applied in engineering if a three-dimensional half-space ballast model is adopted.

According to the ballast model as shown in Fig. 2, the vibrating mass of ballast under a sleeper support point could be evaluated as

$$M_b = \rho_b h_b \left[l_e l_b + (l_e + l_b) h_b \, tg\, \alpha + \tfrac{4}{3} h_b^2 \, tg^2\, \alpha \right], \tag{1}$$

where ρ_b is the density of ballast, h_b is the depth of ballast, l_e is the effective supporting length of half sleeper, l_b is the width of sleeper underside, and α is the ballast stress distribution angle.

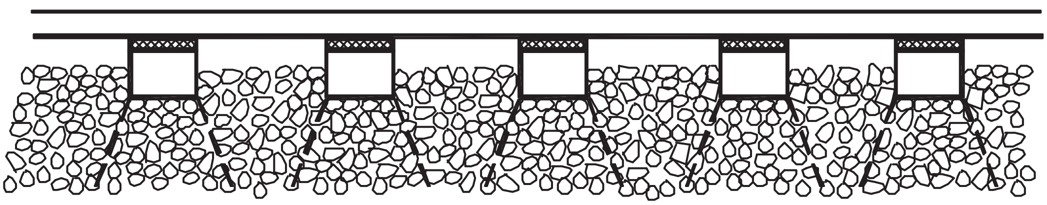

Fig. 1. Load distribution region in continuous granular ballast.

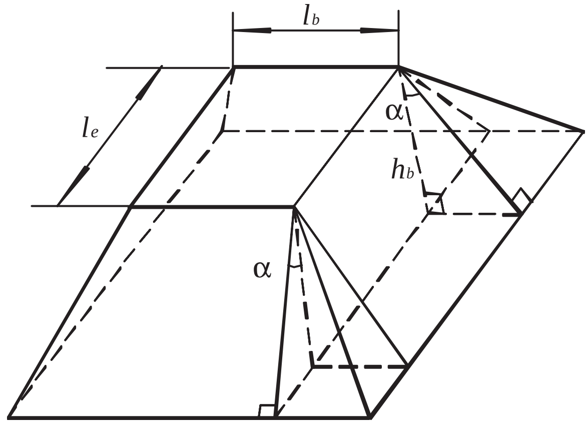

Fig. 2. Model of the ballast under one rail support point.

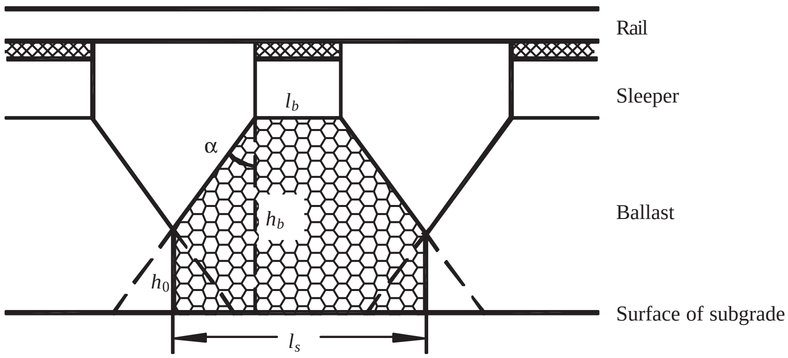

Fig. 3. The modified model of ballast.

The supporting stiffness of a ballast mass can be determined as

$$K_b = \frac{2(l_e - l_b)tg\,\alpha}{\ln[(l_e/l_b)\cdot(l_b + 2h_b\,tg\,\alpha)/(l_e + 2h_b\,tg\,\alpha)]}E_b, \tag{2}$$

where E_b is the elastic modulus of the ballast.

Correspondingly, the subgrade stiffness under one supporting point equals to the product of the cone underside area and the modulus of subgrade:

$$K_f = (l_e + 2h_b\,tg\,\alpha)(l_b + 2h_b\,tg\,\alpha)E_f, \tag{3}$$

where E_f is the K_{30} modulus of subgrade, which means the force acting on unit area that leads to unit deformation.

The established ballast model is based on the assumption that there is no overlapping of adjacent cone regions of ballasts. In the case of thick ballast layer, small sleeper spacing, or big distribution angle, an overlapping of adjacent ballast masses may occur, see Fig. 3. The above ballast model should be modified appropriately. In this case, the vibrating mass of ballast under a rail support point could be defined as the shadowed region as shown in Fig. 3.

The height of the overlapping regions is calculated by

$$h_0 = h_b - \frac{l_s - l_b}{2\, tg\, \alpha}, \tag{4}$$

where l_s is the sleeper spacing.

The ballast vibrating mass is changed into

$$M_b' = \rho_b[l_b h_b(l_e + h_b\, tg\, \alpha) + l_e(h_b^2 - h_0^2)tg\, \alpha + \tfrac{4}{3}(h_b^3 - h_0^3)tg^2\, \alpha]. \tag{5}$$

The ballast supporting stiffness is the combined stiffness of two parts in series

$$K_b' = \frac{K_{b1}K_{b2}}{K_{b1} + K_{b2}}, \tag{6}$$

where

$$K_{b1} = \frac{2(l_e - l_b)tg\, \alpha}{\ln[(l_e l_s)/(l_b(l_e + l_s - l_b))]}E_b, \tag{7}$$

and

$$K_{b2} = \frac{l_s(l_s - l_b + 2l_e + 2h_b\, tg\, \alpha)tg\, \alpha}{l_b - l_s + 2h_b\, tg\, \alpha}E_b. \tag{8}$$

And the subgrade stiffness becomes

$$E_f' = l_s(l_e + 2h_b\, tg\, \alpha)E_f. \tag{9}$$

It can be summarized, when $h_0 \leqslant 0$, the ballast model as shown in Fig. 2 and the formulae (1)–(3) could be used, and when $h_0 > 0$, the modified model as shown in Fig. 3 and the formulae (5)–(9) should be adopted.

In order to account for the continuity and the coupling effects of the interlocking ballast granules, a couple of shear stiffness (K_w) and shear damping (C_w) is introduced between adjacent ballast masses in the ballast model. A complete track dynamic model can then be established, as shown in Fig. 4. Furthermore, the track model can be coupled with various vehicle models so as to carry out dynamic simulations of train-track interactive systems [11,12]. A fast time integration method, however, should be employed to numerically solve the dynamic responses of so large and complicated systems [13]. In actual calculation, the rail is usually considered to be a finitely long beam. In order to minimize the boundary effects, the rail beam should be sufficiently long. In the simulation, the effective length of the rail is determined by the numerical trials. Results show that very good convergence of solution can be obtained if the distance between the moving wheelset and the end of the beam is not less than 15 m. Therefore, the effective length of the rail beam is usually not less than 100 m when the beam is subjected to a moving vehicle model, which is enough long for most vertical excitations arising in the wheel/rail contact. In the case of the moving irregularity model in which the vehicle remains in a fixed position on the rail [2], the effective length of the beam could be reduced to be about 50 m [11].

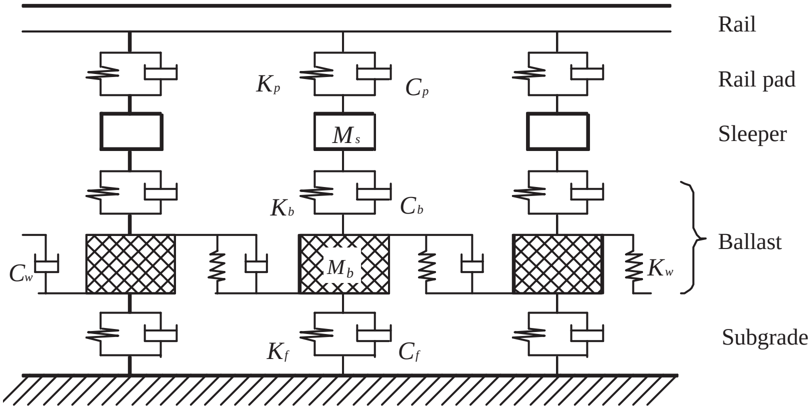

Fig. 4. A complete model for the dynamic analysis of the track.

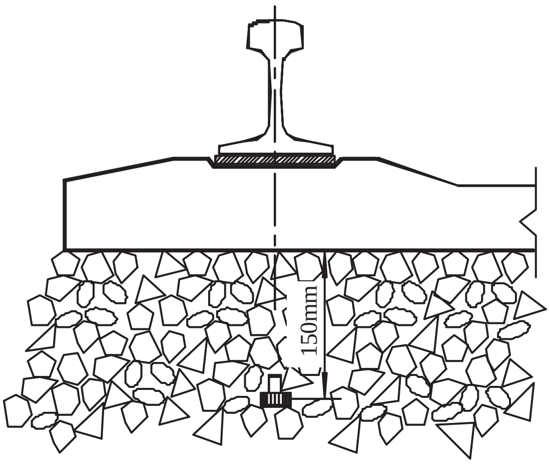

Fig. 5. The position of the measurement point of the ballast acceleration.

3. Field experiment of ballast vibration

In order to verify the ballast vibration model, a full-scale field experiment was carried out in October 2001 at a rail joint on Chengdu–Kunming railway line in Southwest China. The track at the test section consists of the 60 kg/m continuous welded rails, the 60–11 type rubber pads under the rails, the 69-type concrete mono-block sleepers on the ballast-bed of macadam with thickness of 450 mm. A geometric irregularity in cosine wave has been observed on the rail surface of the joint. The wavelength and the wave-depth of the irregularity are approximately 80 and 0.4 mm, respectively. This irregularity becomes a serious impact excitation source when a loaded wheel passes through the joint.

An accelerometer was embedded in the ballast to measure the ballast acceleration, as shown in Fig. 5. A special box was designed to avoid the damage of the accelerometer due to its direct contact with the ballast granules. The outline size of the box is very similar to the ballast granules

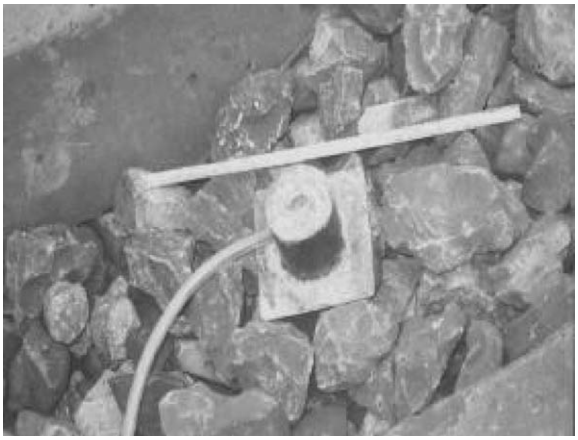

Fig. 6. Special box for the accelerometer in ballast.

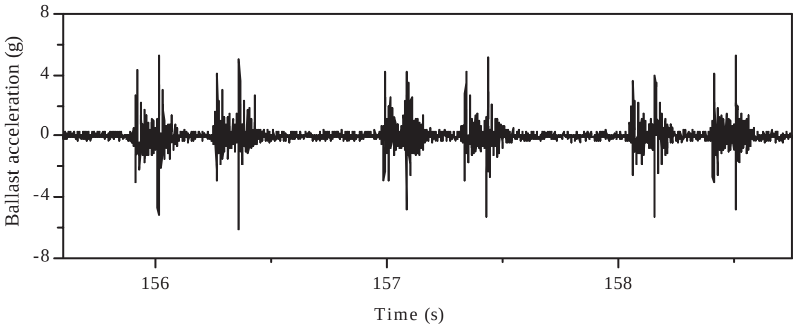

Fig. 7. Example of the ballast acceleration responses due to a passing train.

in order to keep the original ballast configuration (see Fig. 6). That is important for measurements to obtain the actual ballast vibration level, and the measured results have shown satisfactory effect.

Dynamic responses of the ballast accelerations excited by passing trains were measured. Fig. 7 shows an example of the measured results when a passenger train passed with a speed of 90 km/h through the test section. Very obvious impact vibration due to each loaded wheel can be observed from Fig. 7.

4. Comparison of theoretical simulated and field measured results

4.1. Ascertainment of the model parameters

Table 1 lists the track parameters used in the simulation for the test section, in which the stiffness of rail pad was measured in laboratory and the ballast and subgrade parameters were determined based upon the ballast model as shown in Fig. 3. According to the measured results

Table 1
The track model parameters

Notation	Parameter	Value (per rail seat)	Unit
E	Elastic modulus of rail	2.059×10^{11}	N/m^2
I	Rail cross-sectional inertia	3.217×10^{-5}	m^4
m_r	Rail mass per unit length	60.64	kg/m
M_s	Sleeper mass (half)	125.5	kg
K_p	Rail pad stiffness	6.5×10^7	N/m
C_p	Rail pad damping	7.5×10^4	N s/m
l_s	Sleeper spacing	0.545	m
l_e	Effective support length of half sleeper	0.95	m
l_b	Sleeper width	0.273	m
ρ_b	Ballast density	1.8×10^3	kg/m^3
E_b	Elastic modulus of ballast	1.1×10^8	Pa
C_b	Ballast damping	5.88×10^4	N s/m
K_w	Ballast shear stiffness	7.84×10^7	N/m
C_w	Ballast shear damping	8.0×10^4	N s/m
α	Ballast stress distribution angle	35	$^\circ$
h_b	Ballast thickness	0.45	m
E_f	Subgrade K_{30} modulus	9.0×10^7	Pa/m
C_f	Subgrade damping	3.115×10^4	N s/m

for Chinese railway tracks [14], the density of the stable ballast is about 1800 kg/m^3, the elastic modulus of ballast is about 110 MPa, and the K_{30} modulus of subgrade is approximately 90 MPa/m. The ballast vibrating mass under one supporting point can then be calculated from Eq. (5), which is 531.4 kg. The stiffnesses of the ballast and the subgrade are 137.75 and 77.5 MN/m, respectively, calculated from Eqs. (6) and (9).

It should be noted that the ballast stiffness value determined by the present ballast model is located in the range of measured results, 110–185 MN/m, obtained in a full-scale ballasted track model with 400–500 mm thickness ballast at Railway Institute of Tongji University, Shanghai, China. Furthermore, the calculated ballast stiffness value is close to the field-measured result obtained by China Academy of Railway Sciences (CARS), which is about 140 MN/m for Chinese normal 60 kg/m rail tracks [14]. These indicate that the proposed ballast model could give reasonable value of the ballast supporting stiffness, which is one of the most important parameters in the simulation of the ballast vibrations.

It is difficult to determine the ballast damping parameter C_b, which was identified before from the results of the so-called "wheelset-dropping test", originally designed by Sato in Japanese Railways (JR) and now widely used in CR. The ballast shearing parameters K_w and C_w were chosen based upon previous experience data used in CR [11,15], which have not been measured in this experiment.

4.2. Comparison between theoretical and measured results

Fig. 8(a) gives an example of the measured time history of the ballast acceleration when the Chinese main type freight car, C_{62A}, passes at a speed of 60 km/h through the test section. The

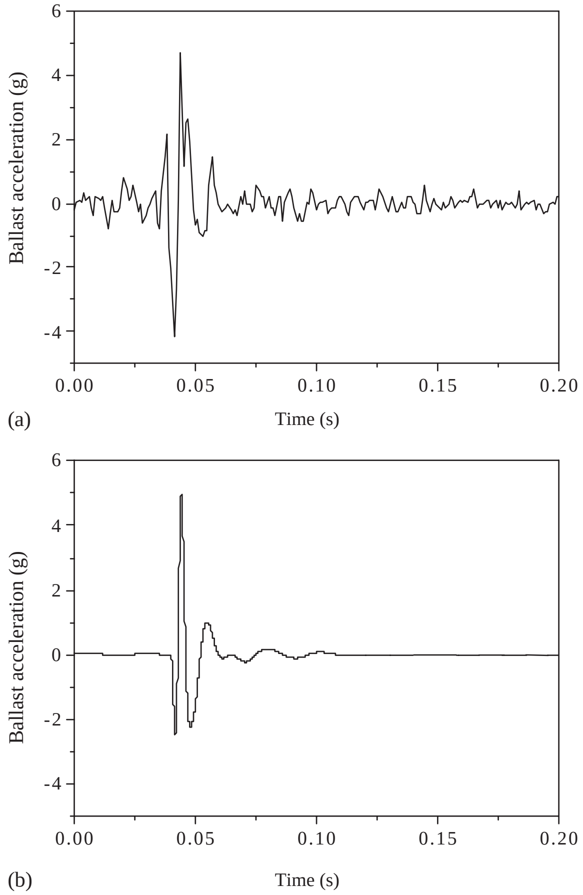

Fig. 8. Comparison of time histories of the ballast accelerations between (a) experiment and (b) theory.

corresponding calculated result with the same running conditions is given in Fig. 8(b). It is shown that the measured and the simulated time responses are rather similar and the maximum accelerations, which are 4.69 g (measured) and 4.97 g (calculated) respectively, are very close to each other.

Fig. 9 gives the corresponding frequency spectra of the ballast accelerations derived from FFT transformation. There is good correlation between the calculated and the measured characteristics of the spectra of the ballast vibration. The calculated resonance frequency range of the ballast acceleration is within 70–100 Hz, whereas the measured result is approximately at 80–110 Hz.

4.3. Influence of shear stiffness and shear damping of the ballast

Since the ballast shearing effect was considered in Refs. [6,15], the shear spring and shear damper have been used in the simulations of train/track dynamic interactions by Ripke et al. [7], Oscarsson et al. [8,9], Igeland et al. [16], and Knothe [3]. However, the necessity of incorporating

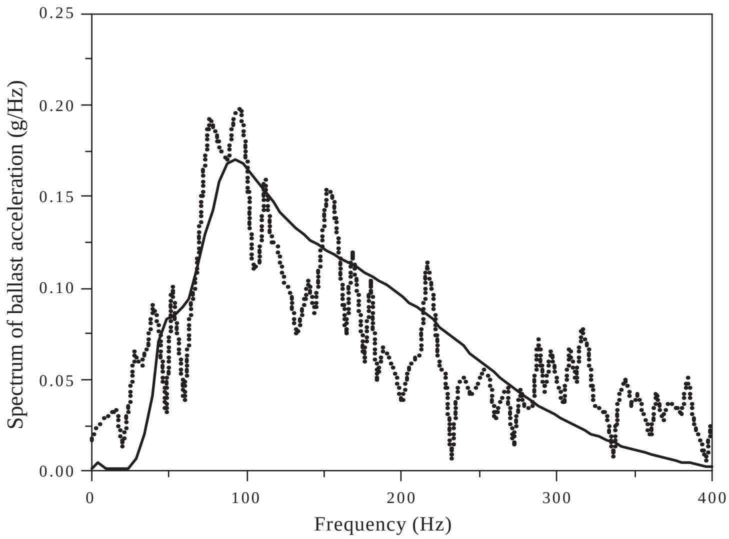

Fig. 9. Comparison of frequency spectra of the ballast accelerations between experiment (------) and theory (——).

Table 2
The ballast shearing effect

Condition of comparison	Measured value	Calculated results without shearing effect	Results with shearing effect	
			Parameters used in the paper	Parameters used in Refs. [8,9]
Ballast acceleration (g)	4.69	5.26	4.97	3.94
Deviation relative to the measured value	—	+12%	+6%	−16%

the shear components into the ballast model has not been well analyzed. How much do they influence the ballast vibration? This will be discussed in the following text.

At first, the ballast accelerations are calculated and compared with and without consideration of the ballast shear effect, see Table 2. Results show that the model will overestimate the ballast vibration level if the ballast shearing effect is not considered—usually the acceleration of the ballast will be at least 10% higher. Secondly, the results are calculated and compared for different values of the ballast shear parameters, see also Table 2. If the values of the ballast shear parameters obtained in Swedish Railways ($K_w = 717\,\mathrm{MN/m}$, $C_w = 173\,\mathrm{kN\,s/m}$) [8,9] are used, the calculated ballast acceleration will be 16% less than the measured one. Meanwhile, the calculated result will be 6% larger than the measured result if the ballast shearing parameters used in Chinese Railways ($K_w = 78.4\,\mathrm{MN/m}$, $C_w = 80\,\mathrm{kN\,s/m}$) [6,11,15] are adopted.

The effect of friction and impact of ballast stones induces a counteracting motion of adjacent ballast blocks, so that the vibration level of one ballast block will be attenuated by the adjacent blocks. If the shearing effect is not considered, this attenuating effect is absent. Thus the ballast mass can vibrate more freely and its vibration level will be overestimated. On the other hand, if higher shear stiffness and shear damping of the ballast are adopted, it implies that the stronger attenuating effect between neighboring ballast blocks is considered, which results in less ballast vibration. It can be concluded that the ballast shearing parameters have great influence on the dynamic behavior of tracks and strongly depend on the structure and the material of the ballast.

5. Conclusions

A method of modelling the ballast vibration has been presented in this article. A five-parameter ballast vibration model has been established on the basis of the hypothesis that the transmission of the load from a sleeper to the ballast approximately coincides with a cone distribution. Effects of shear stiffness and shear damping of ballast have been taken into account in the ballast model. A full-scale field experiment was carried out to measure the ballast vibrations. Theoretical simulation results agree well with the measured results. Hence the proposed ballast vibration model has been validated.

It is necessary for the analysis of track dynamics to consider the ballast shearing effect between adjacent ballast masses to model the continuity and the coupling effects of the interlocking ballast granules.

The ballast vibrates mainly in the mid-frequency range. The resonance frequency range of the ballast vibration is at 70–100 Hz by calculation and at 80–110 Hz by measurement, respectively.

Acknowledgements

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